Closed-Form Semi-Blind Receiver For MIMO Relay Systems Using Double Khatri–Rao Space-Time Coding

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***Abstract*—In this letter, we consider a one-way two-hop AF relaying scheme employing two independent Khatri-Rao space-time (KRST) codings at the source and relay nodes. The signals received at destination form a fourth-order tensor whose dimensions correspond to four signal diversities, and which satisfies a nested PARAFAC model. Exploiting this nested structure, we derive two matrix unfoldings expressed in terms of two Khatri-Rao products which are used to propose a closed-form semi-blind receiver allowing to jointly estimate the information symbols and the individual channels. A numerical analysis shows that this new receiver achieves a substantial computational complexity reduction over an iterative (ALS-based) semi-blind receiver, especially in presence of a great number of source and/or relay antennas.**

***Index Terms*—Channel estimation, cooperative relaying, KRST coding, nested PARAFAC, semi-blind receiver.**

I. Introduction

THE use of relay stations between the source and destination nodes of a MIMO wireless communication system is a pertinent technique to mitigate some propagation issues, such as path loss and shadowing [1], [2]. Several works have pointed out that the channel state information (CSI) of both *source-relay* and *relay-destination* links is highly desirable for optimizing two-hop MIMO relay systems [3]–[7]. With non-regenerative protocols, such as the amplify-and-forward (AF) one, the concatenation of two transmission hops without decoding at the relay implies that conventional point-to-point estimation strategies cannot dissociate the channels of both links, and consequently the efficiency of optimization techniques is compromised.

Unlike point-to-point MIMO systems for which several tensor-based receivers have been proposed [8]–[12], few works have addressed tensor-based methods to solve the problem of channel estimation in MIMO AF relay systems. Most of these works use pilot symbols, which induces a loss of spectral efficiency [13]–[15].

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Recently, two alternative strategies were proposed to jointly estimate the information symbols and the individual channels in a one-way two-hop AF relaying system [16], [17]. In [16], three semi-blind receivers are proposed using a simplified Khatri-Rao space-time (KRST) coding [9] at the source node. The signals received at the destination node then satisfy a PARATUCK2 model [18]. In [17], the authors propose a similar transmission scheme with a KRST coding at the relay, which introduces an extra time diversity with respect to the system proposed in [16], leading to a nested PARAFAC model [19] for the tensor of signals received at destination. This supplementary time diversity is at the origin of a remarkable improvement of the bit error rate (BER) performance. A double two-step alternating least squares receiver, called DALS, was derived in exploiting the nested PARAFAC model. This receiver is composed of two ALS-based algorithms, denoted ALS-X and ALS-Z, which are dedicated respectively to symbol and channel estimation. Even if competitive, this receiver presents the drawbacks to be iterative and to need several matrix right inversions that limit its effectiveness. This letter proposes a closed-form (SVD-based) semi-blind receiver which relies on a double Khatri-Rao factorization (DKRF). This new receiver offers the same performance as the DALS one proposed in [17] with the great practical advantage of being non-iterative, which entails a substantial complexity reduction as demonstrated by our numerical analysis.

Notations: Scalars, column vectors, matrices, and tensors are denoted by lower-case (x), boldface lower-case ($\mathbf{x}$), boldface capital ($\mathbf{X}$), and calligraphic ($\mathcal{X}$) letters, respectively. $\mathbf{X}^T$, $\mathbf{X}^*$, $\mathbf{X}^\dagger$, $\mathbf{X}_{l\cdot}$ and $\mathbf{X}_{\cdot m}$ are the transpose, the conjugate, the pseudoinverse, the lth row, and the mth column of $\mathbf{X} \in \mathbb{C}^{L\times M}$, respectively. $D_n(\mathbf{X})$ stands for the diagonal matrix formed from the elements of $\mathbf{X}_{n\cdot}$. Given a third-order tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$, with entry $x_{i,j,k}$, the matrices $\mathbf{X}_{JK\times I}$, $\mathbf{X}_{KI\times J}$ and $\mathbf{X}_{IJ\times K}$ denote tall mode-1, mode-2 and mode-3 unfoldings, with $x_{i,j,k} = [\mathbf{X}_{JK\times I}]_{(k-1)J+j,i} = [\mathbf{X}_{KI\times J}]_{(i-1)K+k,j} = [\mathbf{X}_{IJ\times K}]_{(j-1)I+i,k}$. The vec and unvec operators are defined by $\mathbf{x}_{JKI} = \text{vec}(\mathbf{X}_{JK\times I}) \in \mathbb{C}^{JKI\times 1} \leftrightarrow \mathbf{X}_{JK\times I} = \text{unvec}(\mathbf{x}_{JKI})$.

A PARAFAC decomposition [20] of a third-order tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$, with rank-$R$ and matrix factors $(\mathbf{A}, \mathbf{B}, \mathbf{C})$, will be noted $\|\mathbf{A}, \mathbf{B}, \mathbf{C}; R\|$. Tall and flat mode-1 matrix unfoldings of $\mathcal{X}$ are respectively given by

$$\mathbf{X}_{JK\times I} = (\mathbf{C} \diamond \mathbf{B})\mathbf{A}^T = (\mathbf{X}_{I\times JK})^T, \qquad (1)$$

where $\diamond$ denotes the Khatri-Rao product.

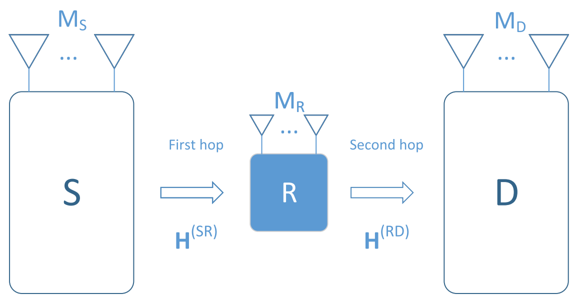

Fig. 1. One-way two-hop relay system model.

II. System Model

We consider a one-way relay MIMO system, where the communication is divided into two hops (Fig. 1). During the first one, the source node transmits the information symbols to the relay (SR link), and during the second one the source stays silent, while the relay forwards amplified signals to the destination node (RD link). M_D, M_S and M_R are the numbers of antennas at the destination, source and relay nodes, respectively. The communication channels are considered flat-fading and invariant during the transmission protocol.

Let $\mathbf{S} \in \mathbb{C}^{N\times M_S}$ be a matrix containing N data-streams composed of M_S symbols which are multiplexed onto the M_S source antennas, all symbols of the first data-stream being equal to one. A simplified Khatri-Rao space-time (KRST) coding [9]– i.e. without the linear constellation precoding (LCP) matrix–is used at the source for introducing temporal redundancy with the code matrix $\mathbf{C} \in \mathbb{C}^{P\times M_S}$, where P is the source (spreading) code length, such that the pth repetition of the nth coded data-stream, allocated to the m_Sth antenna, satisfies the following equation

$$\bar{s}_{m_S,p,n} = c_{p,m_S} s_{n,m_s}. \tag{2}$$

The third-order tensor $\bar{\mathcal{S}} \in \mathbb{C}^{M_S\times P\times N}$ contains the coded signals to be sent by the source. The noiseless signals received by the relay after transmission of the coded signals via the *source-relay* channel $\mathbf{H}^{(SR)} \in \mathbb{C}^{M_R\times M_S}$ are given by

$$\begin{aligned} w_{m_R,p,n} &= \sum_{m_S=1}^{M_S} h^{(SR)}_{m_R,m_S} \bar{s}_{m_S,p,n} \\ &= \sum_{m_S=1}^{M_S} h^{(SR)}_{m_R,m_S} c_{p,m_s} s_{n,m_s}. \end{aligned} \tag{3}$$

The set of signals given by (3) forms a third-order tensor $\mathcal{W} \in \mathbb{C}^{M_R\times P\times N}$ satisfying a PARAFAC decomposition $\|\mathbf{H}^{(SR)}, \mathbf{C}, \mathbf{S}; M_S\|$. At the relay, a second KRST coding is performed on the incoming signals, delivering the following coded signals

$$\bar{w}_{m_R,j,p,n} = g_{j,m_R} w_{m_R,p,n}, \tag{4}$$

where $\mathbf{G} \in \mathbb{C}^{J\times M_R}$ is the relay code matrix, and J is its code length. This second KRST coding introduces a supplementary time spreading. The relaying protocol is non-regenerative, in the sense that no decoding is performed at the relay. The coded signals are forwarded through the *relay-destination* channel $\mathbf{H}^{(RD)} \in \mathbb{C}^{M_D\times M_R}$. Using (3) and (4), the signals received at destination in absence of noise are given by

$$\begin{aligned} x_{m_D,j,p,n} &= \sum_{m_R=1}^{M_R} h^{(RD)}_{m_D,m_R} \bar{w}_{m_R,j,p,n}, \\ &= \sum_{m_R=1}^{M_R} \sum_{m_S=1}^{M_S} h^{(RD)}_{m_D,m_R} g_{j,m_R} h^{(SR)}_{m_R,m_S} c_{p,m_S} s_{n,m_s}, \end{aligned} \tag{5}$$

$$= \sum_{m_S=1}^{M_S} z_{m_D,j,m_S} c_{p,m_S} s_{n,m_s}, \tag{6}$$

where $\mathcal{Z} \in \mathbb{C}^{M_D\times J\times M_S}$ is the effective channel tensor which satisfies the PARAFAC model $\|\mathbf{H}^{(RD)}, \mathbf{G}, (\mathbf{H}^{(SR)})^T; M_R\|$, i.e.

$$z_{m_D,j,m_S} = \sum_{m_R=1}^{M_R} h^{(RD)}_{m_D,m_R} g_{j,m_R} h^{(SR)}_{m_R,m_S}. \tag{7}$$

Eq. (6) defines a fourth-order tensor $\mathcal{X} \in \mathbb{C}^{M_D\times J\times P\times N}$ for the signals received at destination. This tensor satisfies a nested PARAFAC model (cf. (37), [17]). By combining the first two modes of $\mathcal{X}$, this tensor can be reformulated as the PARAFAC model $\|\mathbf{Z}_{M_D J\times M_S}, \mathbf{C}, \mathbf{S}; M_S\|$. From this PARAFAC model and Eq. (7), one can deduce the following matrix unfoldings of the tensors $\mathcal{X}$ and $\mathcal{Z}$

$$\mathbf{X}_{NM_D J\times P} = (\mathbf{Z}_{M_D J\times M_S} \diamond \mathbf{S})\, \mathbf{C}^T, \tag{8}$$

$$\mathbf{Z}_{M_S M_D\times J} = \left(\mathbf{H}^{(RD)} \diamond \left(\mathbf{H}^{(SR)}\right)^T\right) \mathbf{G}^T. \tag{9}$$

These two equations written in terms of Khatri-Rao products are at the basis of the *double Khatri-Rao factorization* (DKRF) based receiver presented in the next section.

III. Semi-Blind DKRF Receiver

Given the tensor $\mathcal{X}$ of signals received at destination, the proposed DKRF receiver jointly estimates the symbol matrix $\mathbf{S}$ and the individual channels $\mathbf{H}^{(RD)}$ and $\mathbf{H}^{(SR)}$ by means of (8) and (9). Assume that the code matrices $\mathbf{C}$ and $\mathbf{G}$ are known at the receiver and designed with orthonormal columns, which implies $\mathbf{C}^T\mathbf{C}^* = \mathbf{I}_{M_S}$ and $\mathbf{G}^T\mathbf{G}^* = \mathbf{I}_{M_R}$. Then, let us define

$$\mathbf{A} = \mathbf{Z}_{M_D J\times M_S} \diamond \mathbf{S}, \tag{10}$$

$$\mathbf{B} = \mathbf{H}^{(RD)} \diamond (\mathbf{H}^{(SR)})^T. \tag{11}$$

Denoting $\widetilde{\mathcal{X}}$ the tensor of noisy signals received at destination, and $\widehat{\mathbf{Z}}_{M_S M_D\times J}$ an estimate of $\mathbf{Z}_{M_S M_D\times J}$ deduced by reshaping the estimate $\widehat{\mathbf{Z}}_{M_D J\times M_S}$, the LS estimates of the Khatri-Rao products $\mathbf{A}$ and $\mathbf{B}$ are deduced from (8) and (9) as

$$\widehat{\mathbf{A}} = \widetilde{\mathbf{X}}_{NM_D J\times P}\mathbf{C}^* \tag{12}$$

$$\widehat{\mathbf{B}} = \widehat{\mathbf{Z}}_{M_S M_D\times J}\mathbf{G}^*. \tag{13}$$

The DKRF receiver is composed of two estimation steps, herein referred to as KRF-X (Alg. 1) and KRF-Z (Alg. 2). These algorithms exploit the LS estimates (12) and (13) of the Khatri-Rao products (10) and (11) for estimating the matrix factors $\mathbf{S}$ and $\mathbf{Z}_{M_D J\times M_S}$, on one hand, and the channels $\mathbf{H}^{(SR)}$ and $\mathbf{H}^{(RD)}$, on the other hand. These algorithms are based on the reorganization of the Khatri-Rao product of two vectors $(\mathbf{u} \diamond \mathbf{v})$ into a rank-one matrix unvec $(\mathbf{u} \diamond \mathbf{v}) = \mathbf{v}\mathbf{u}^T$, implying that the factors of this Khatri-Rao product can be obtained

Algorithm 1. KRF-X: Symbol estimation

Inputs $\widetilde{\mathbf{X}}_{NM_DJ\times P}$ and $\mathbf{C}$

1) Calculate the LS estimate $\widehat{\mathbf{A}} = \widetilde{\mathbf{X}}_{NM_DJ\times P}\mathbf{C}^* \in \mathbb{C}^{NM_DJ\times M_S}$.
2) For $m_S \in \{1, \cdots, M_S\}$:
   i. Reshape $\widehat{\mathbf{A}}_{\cdot m_S}$ into unvec $(\widehat{\mathbf{A}}_{\cdot m_S}) \in \mathbb{C}^{N\times M_DJ}$.
   ii. Compute the rank-one approximation of unvec $(\widehat{\mathbf{A}}_{\cdot m_S})$.
   iii. Deduce $\widehat{\mathbf{S}}_{\cdot m_S} = \mathbf{u}\sqrt{\sigma_{\max}}$ and $(\widehat{\mathbf{Z}}_{M_DJ\times M_S})_{\cdot m_S} = \mathbf{v}^*\sqrt{\sigma_{\max}}$, where $\sigma_{\max}$ is the largest singular value, and $\mathbf{u}$ and $\mathbf{v}$ are respectively the associated left-singular and right-singular vectors.
3) Remove the column scaling ambiguities using (14).

Outputs $\widehat{\mathbf{S}}$ and $\widehat{\mathbf{Z}}_{M_DJ\times M_S}$

Algorithm 2. KRF-Z: Channels estimation

Inputs $\widehat{\mathbf{Z}}_{M_DJ\times M_S}$ and $\mathbf{G}$

1) Reshape $\widehat{\mathbf{Z}}_{M_DJ\times M_S}$ into $\widehat{\mathbf{Z}}_{M_SM_D\times J}$.
2) Calculate the LS estimate $\widehat{\mathbf{B}} = \widehat{\mathbf{Z}}_{M_SM_D\times J}\mathbf{G}^* \in \mathbb{C}^{M_SM_D\times M_R}$.
3) For $m_R \in \{1, \cdots, M_R\}$:
   i. Reshape $\widehat{\mathbf{B}}_{\cdot m_R}$ into unvec $(\widehat{\mathbf{B}}_{\cdot m_R}) \in \mathbb{C}^{M_S\times M_D}$.
   ii. Compute the rank-one approximation of unvec $(\widehat{\mathbf{B}}_{\cdot m_R})$.
   iii. Deduce $\left(\widehat{\mathbf{H}}^{(SR)}_{m_R\cdot}\right)^T = \mathbf{u}\sqrt{\sigma_{\max}}$ and $\widehat{\mathbf{H}}^{(RD)}_{\cdot m_R} = \mathbf{v}^*\sqrt{\sigma_{\max}}$, where $\sigma_{\max}$ is the largest singular value, and $\mathbf{u}$ and $\mathbf{v}$ are respectively the associated left-singular and right-singular vectors.
4) Remove the column scaling ambiguities using (15).

Outputs $\widehat{\mathbf{H}}^{(SR)}$ and $\widehat{\mathbf{H}}^{(RD)}$

by calculating the rank-one approximation of the matrix unvec $(\mathbf{u}\diamond\mathbf{v})$ by means of its SVD. Applying this result to the Khatri-Rao product of two matrices $(\mathbf{U}\diamond\mathbf{V})$, with $\mathbf{U}\in\mathbb{C}^{I\times R}$ and $\mathbf{V}\in\mathbb{C}^{J\times R}$, these matrices can be calculated column by column via the SVD of each rank-one matrix $\mathbf{v}_{.r}\mathbf{u}_{.r}^T$ associated with the Khatri-Rao product of the rth columns of the matrix factors $\mathbf{U}$ and $\mathbf{V}$. This idea was used in [21] for finding the factors of the Kronecker product of two matrices. It was exploited in [22] for estimating two matrix factors of a third-order PARAFAC decomposition when one factor is known. It was also used in [23] for estimating the channels of an AF relaying system.

A. Code Design and Identifiability

As mentioned at the beginning of Section III, the code matrices $\mathbf{C}$ and $\mathbf{G}$ are designed to have orthonormal columns. In the standard KRST coding [9], the code matrix $\mathbf{C}$ is designed as a truncated DFT matrix, which enables a performance selection between full diversity ($P \geq M_S$) and full transmission rate ($P = 1$). Motivated by the same purpose, the same source code matrix is used here. Therefore, since $P \geq M_S$, the DKRF receiver exploits full (transmit) diversity.

The double KRST coding allows a trade-off between source and relay code lengths to obtain better coding gains than the single KRST coding for a same transmission rate. In addition, if one wishes to estimate only the symbols, then KRF-Z becomes optional, and J can take values smaller than M_R to increase the transmission rate rather than the (relay) transmit diversity. However, if estimation of the channels $\mathbf{H}^{(RD)}$ and $\mathbf{H}^{(SR)}$ is needed, then the condition $J \geq M_R$ must be verified.

Regarding the identifiability issue, the right invertibility of $\mathbf{C}^T$ and $\mathbf{G}^T$ implies $P \geq M_S$ and $J \geq M_R$, which are necessary and sufficient identifiability conditions for using the KRF-X and KRF-Z algorithms. Moreover, the column orthonormality of $\mathbf{C}$ and $\mathbf{G}$ allows to simplify the calculation of the right inverses of $\mathbf{C}^T$ and $\mathbf{G}^T$ as $\mathbf{C}^*$ and $\mathbf{G}^*$, respectively. It is worth mentioning that these identifiability conditions are quite simple, being directly linked to the choice of the code matrices $\mathbf{C}$ and $\mathbf{G}$. There is no other additional rank (or dimensionality) condition to be satisfied on the channel and symbol matrices, which is in contrast with [17] where a more complicated set of conditions must be satisfied to ensure identifiability using the NPALS and DALS receivers (please see the five inequalities in Theorem 1, Section IV-C in [17]).

B. Uniqueness Conditions

Under the assumptions that the channels are rich scattering and that the symbol matrix is full column rank, with M_S and $M_R \geq 2$, the nested PARAFAC model is unique under the condition $\min(M_D, M_R) \geq \max(M_R - M_S + 2, 2)$[17]. The column scaling ambiguities in the outputs of KRF-X and KRF-Z can be eliminated by assuming known the first row of $\mathbf{S}$ and $\mathbf{H}^{(RD)}$, and modifying the estimated matrices as follows

$$\widehat{\mathbf{S}} \leftarrow \widehat{\mathbf{S}}\Lambda^{(S)}, \quad \widehat{\mathbf{Z}}_{M_DJ\times M_S} \leftarrow \widehat{\mathbf{Z}}_{M_DJ\times M_S}(\Lambda^{(S)})^{-1}, \tag{14}$$

$$\widehat{\mathbf{H}}^{(RD)} \leftarrow \widehat{\mathbf{H}}^{(RD)}\Lambda^{(H)}, \ \widehat{\mathbf{H}}^{(SR)} \leftarrow (\Lambda^{(H)})^{-1}\widehat{\mathbf{H}}^{(SR)}, \tag{15}$$

$$\Lambda^{(S)} = D_1(\mathbf{S})D_1^{-1}(\widehat{\mathbf{S}}), \Lambda^{(H)} = D_1(\mathbf{H}^{(RD)})D_1^{-1}(\widehat{\mathbf{H}}^{(RD)}).$$

In practice, such a knowledge of the first row of $\mathbf{H}^{(RD)}$ can be obtained by means of a simple LS estimation step using a short training sequence generated by the relay [17].

C. Computational Complexity

In the following table, we compare the computational complexity of the DKRF and DALS receivers by evaluating the dominant cost for SVD calculation needed both to calculate the rank-one approximations for the DKRF receiver, and the right inverses of Khatri-Rao products with the DALS receiver. In this last case, the complexity cost is weighted by l_1 and l_2, which are the average numbers of iterations for convergence of the ALS-X and ALS-Z algorithms, respectively. Note that for a matrix of dimensions $I_1 \times I_2$, the complexity of SVD computation is around $O(I_1I_2\min(I_1, I_2))$ [24].

Define the ratios $O_1 = O_B/O_A$, $O_2 = O_D/O_C$, and $O_3 = (O_B + O_D)/(O_A + O_C)$, which express how many times DALS is more computationally demanding than DKRF for symbol, channels and overall estimation, respectively. We have

	Alg.	Computational cost: $O(.)$
I	KRF-X	$O_A = \min(M_D J, N) M_D J N M_S$
	ALS-X	$O_B = l_1 \left(M_D J + N\right) P M_S^2$
II	KRF-Z	$O_C = \min(M_D, M_S) M_S M_D M_R$
	ALS-Z	$O_D = l_2 \left(M_D + M_S\right) J M_R^2$
I: Symbol estimation; II: Channel estimation l_1,l_2: Number of iterations		

$$O_1 = l_1 P M_S \frac{M_D J + N}{\min(M_D J, N) M_D J N}, \quad (16)$$

$$O_2 = l_2 J M_R \frac{M_D + M_S}{\min(M_D, M_S) M_S M_D}. \quad (17)$$

Note that O_1 is linear with respect to M_S, which means that concerning symbol estimation DALS is more negatively affected than DKRF by an increase of this parameter. Similarly, increasing M_R favors the proposed receiver for channel estimation, since O_2 scales linearly as a function of the number of relay antennas. Besides, since $P \geq M_S$ is an identifiability condition for both receivers, an increase of the code length P to cope with a larger M_S only penalizes DALS. Similarly, an increase of J penalizes ALS-Z but not KRF-Z.

In contrast, for symbol estimation, the DKRF receiver behaves worse with larger values of N and $M_D J$, since its complexity scales linearly with their product rather than their sum. In summary, the DKRF receiver is definitely preferable for systems with moderate to large numbers of antennas at the source and/or the relay. Another great advantage of DKRF is to be non-iterative unlike DALS, as shown by the factors l_1 and l_2 in O_1 and O_2.

IV. Simulation Results and Discussion

In this section, the DKRF and DALS receivers are compared in terms of computational complexity by means of Monte Carlo simulations. The curves were obtained as an average over 5×10^4 Monte Carlo runs. In all simulations, $P = J = M_D = N = 8$ are used to satisfy all identifiability and uniqueness conditions. The source and relay coding matrices $\mathbf{C}$ and $\mathbf{G}$ are (truncated) DFT matrices, and we assume $\mathbf{H}^{(SR)} \sim \mathcal{CN}(0, 1/M_S)$ and $\mathbf{H}^{(RD)} \sim \mathcal{CN}(0, 1/M_R)$. The symbol matrix is expressed as $\mathbf{S} = \sqrt{E_S}\mathbf{S}_o$, where $\mathbf{S}_o$ contains unit-power symbols randomly drawn from a 8-PSK alphabet, and E_S denotes the symbol energy. The first row of $\mathbf{S}$ and $\mathbf{H}^{(RD)}$ is composed of ones to avoid scaling ambiguities (see §III-B). The additive noise samples at relay and destination are drawn from a complex Gaussian distribution with zero mean and unit variance. For DALS, random initializations are used at each run and the stop criterion is based on the normalized reconstruction error between two successive iterations [17]. For all simulations, DKRF and DALS have provided nearly the same performance in terms of BER and channel NMSE.

Fig. 2 shows the complexity ratios O_1, O_2 and O_3, for $E_S = 10$ dB, calculated using average values for l_1 and l_2 obtained from all the Monte Carlo runs. From this figure, we can note that DKRF is much less computationally demanding than DALS (for $M_S = 2$, we have $O_3 = 3$ meaning that DKRF is nearly 3 times less costly than DALS). When the number of antennas at the source and/or relay increases, the complexity savings with DKRF are even more pronounced. Note that,

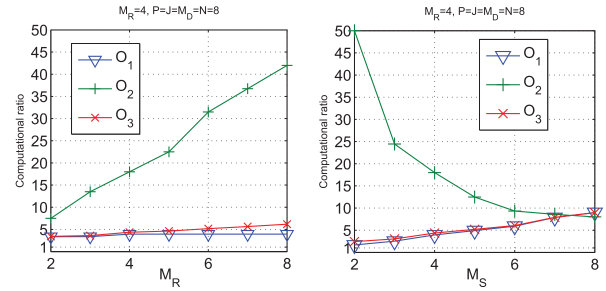

Fig. 2. Complexity ratio between DALS and DKRF.

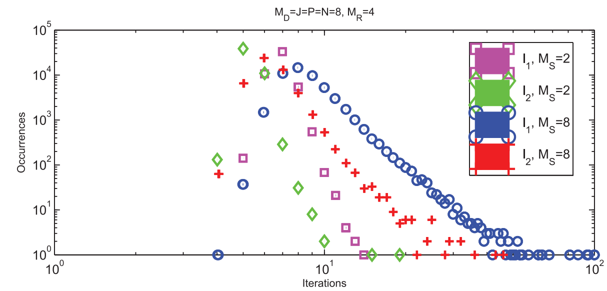

Fig. 3. Histogram of l_1 and l_2.

although O_2 decreases as a function of M_S, the complexity associated with symbol estimation dominates the overall cost of the receivers, i.e. $O_A \gg O_C$ and $O_B \gg O_D$, and O_3 is close to O_1, with $O_1 = O_3$ when only symbol estimation is carried out. The dominance of the symbol estimation step usually happens when the number N of data streams is large compared to the number of relay/source antennas. As M_R increases, the DKRF receiver becomes more attractive than the DALS one due to a linear increase of O_2 as a function of M_R. More generally, when large arrays are used at the source and relay stations (large values of M_S and M_R), the complexity gains of DRKF over DALS become more significant.

Fig. 3 plots an histogram of the number of iterations l_1 and l_2 with DALS, for $M_S = 2$ and $M_S = 8$. The other system parameters are those considered in Fig. 2. One can note that, in most of the occurrences, the required number of iterations for convergence fall within 4 and 10, which shows an *a priori* good behavior of DALS. However, in the worst case scenarios for DALS, l_1 and l_2 can be an order of magnitude greater than their mean values used to plot Fig. 2, which further increases the complexity ratios between DALS and DKRF.

To conclude, DKRF is much more computationally efficient than DALS, while offering the same performance. The complexity of the latter is always a function of the number of iterations, which depends not only on the initialization but also on the dimensions of the system. Moreover, the DKRF can benefit from parallel processing since the rank-one approximations in each algorithm KRF-X and KRF-Z can be calculated in parallel. It is worth mentioning that MMSE-based estimators which exploit the second-order statistics of the noise can also be used to refine the channel and symbol estimates. These estimators can be considered as either a pre-processing (before the DKRF algorithm) or a post-processing (after the DKRF algorithm) stage. Finally, the performance of the proposed closed-form receiver can be further improved to deal with colored (Kronecker-structured) noise by means of tensor pre-whitening techniques [25], [26].

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