Double Khatri-Rao Space-Time-Frequency Coding Using Semi-Blind PARAFAC Based Receiver

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Abstract

In this letter, we first introduce a new class of tensor models for fourth-order tensors, referred to as "nested PARAFAC models". Then, we present a space-time-frequency (STF) coding scheme for multiple antenna orthogonal frequency division multiplexing systems. The proposed scheme, called double Khatri-Rao STF (D-KRSTF) coding, combines time-domain spreading with space-frequency precoding and provides an extension of Khatri-Rao space-time (KRST) coding [1]. We show that the received signals define a fourth-order tensor satisfying two nested PARAFAC models, and a semi-blind receiver is then derived using a two-step alternating least squares algorithm for joint channel and symbol estimation. Simulation results show that our semi-blind receiver offers superior performance compared with some previously proposed tensor-based solutions and operates close to the zero forcing receiver with perfect channel state information.

Index Terms

Space-time-frequency codes, MIMO systems, Khatri-Rao product, nested PARAFAC models.

EDICS: COM-ESTI, COM-MIMO.

I. Introduction

A number of space-time coding methods with blind detection have been developed using tensor models [1]- [5]. The approach of [1] proposes a blind-decodable space-time coding based on the parallel factor

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(PARAFAC) decomposition [6]. The work [2] is based on the same approach as [1] but for frequency selective channels. In [3], a trilinear space-time-frequency coding structure is presented, while in [4], the so-called constrained factor (CONFAC) model is proposed to derive a wider class of space-time coding schemes compared to the previous tensor-based works. Recently, a space-time coding relying on the PARATUCK2 tensor model has been proposed [5], which allows both spreading and multiplexing of data symbols across space and time. More recently, this model has been generalized to space-time-frequency coding [7].

Matrix-based space-time/space-frequency coding methods for orthogonal frequency division multiplexing (OFDM) systems with blind or semi-blind detection have been proposed in the past few years in a number of works (see e.g. [8], [9] and references therein). The existing matrix-based solutions rely either on computationally demanding maximum likelihood (ML) detection strategies or on lower complexity detection strategies. For instance, the method [8] is based on a semi-definite relaxation approach while [9] relies to second-order statistics of the data.

In this letter, we propose a new class of tensor models for fourth-order tensors that we call nested PARAFAC models. Then, we present a new space-time-frequency (STF) coding scheme for multiantenna OFDM systems. This scheme, referred to as double Khatri-Rao space-time-frequency (D-KRSTF) coding, combines time-domain spreading with a space-frequency constellation rotation (CR) precoding. We show that the received signals define a fourth-order tensor that satisfies two nested PARAFAC models. By exploiting two different ways of nesting the underlying third-order PARAFAC models, we derive a semi-blind receiver based on a two-step alternating least squares algorithm for joint channel and symbol estimation.

In contrast to matrix-based decoding methods such as those in [8], [9], which exploit either the space-time or space-frequency codeword structure, the proposed receiver capitalizes on the tensorial structure of the joint space-time-frequency codeword to operate semi-blindly with fewer receive antennas than transmit antennas. As shown in our simulation results, the proposed transceiver has a simpler code design and yields superior performance in comparison with existing tensor-based schemes and receivers. Moreover, it does not require constant-energy constellations as in differential schemes.

Notations: Scalars are denoted by lower-case letters $(a, b, \ldots)$, vectors by boldface lower-case letters $(\mathbf{a}, \mathbf{b}, \ldots)$, matrices by boldface capitals $(\mathbf{A}, \mathbf{B}, \ldots)$, and tensors by calligraphic letters $(\mathcal{A}, \mathcal{B}, \ldots)$. $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ stand for transpose and pseudo-inverse of $\mathbf{A}$, respectively. $\mathbf{A}_{i.} \in \mathbb{C}^{1\times R}$ denotes the i-th row of $\mathbf{A} \in \mathbb{C}^{I\times R}$. The operator diag$(\mathbf{a})$ forms a diagonal matrix from its vector argument, while $D_i(\mathbf{A})$ constructs a diagonal matrix out of the i-th row of $\mathbf{A}$. The Khatri-Rao product between $\mathbf{A} \in \mathbb{C}^{I\times R}$ and $\mathbf{B} \in \mathbb{C}^{J\times R}$ is given by $\mathbf{A} \diamond \mathbf{B} = [\mathbf{A}_{.1} \otimes \mathbf{B}_{.1}, \ldots, \mathbf{A}_{.R} \otimes \mathbf{B}_{.R}] \in \mathbb{C}^{IJ\times R}$.

II. Nested PARAFAC models

Let us consider the following model for the fourth-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times J_1 \times I_2 \times J_2}$

$$x_{i_1,j_1,i_2,j_2} = \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} a_{i_1,r_1}^{(1)} b_{j_1,r_1}^{(1)} a_{i_2,r_2}^{(2)} b_{j_2,r_2}^{(2)} g_{r_1,r_2}. \quad (1)$$

This model can be interpreted as two nested third-order PARAFAC models sharing $\mathbf{G} \in \mathbb{C}^{R_1 \times R_2}$ as a common matrix factor. Indeed, let us define the third-order tensors $\mathcal{Z}^{(1)} \in \mathbb{C}^{I_1 \times J_1 \times R_2}$ and $\mathcal{Z}^{(2)} \in \mathbb{C}^{I_2 \times J_2 \times R_1}$ such as

$$z_{i_1,j_1,r_2}^{(1)} = \sum_{r_1=1}^{R_1} a_{i_1,r_1}^{(1)} b_{j_1,r_1}^{(1)} g_{r_1,r_2} \quad (2)$$

$$z_{i_2,j_2,r_1}^{(2)} = \sum_{r_2=1}^{R_2} a_{i_2,r_2}^{(2)} b_{j_2,r_2}^{(2)} g_{r_1,r_2} \quad (3)$$

Equations (2) and (3) correspond to PARAFAC decompositions of the tensors $\mathcal{Z}^{(1)}$ and $\mathcal{Z}^{(2)}$, with matrix factors $(\mathbf{A}^{(1)}, \mathbf{B}^{(1)}, \mathbf{G}^T)$ and $(\mathbf{A}^{(2)}, \mathbf{B}^{(2)}, \mathbf{G})$, respectively. These tensors admit the following unfolded matrix forms

$$\mathbf{Z}^{(n)} = (\mathbf{A}^{(n)} \diamond \mathbf{B}^{(n)}) \mathbf{C}^{(n)T} \in \mathbb{C}^{K_n \times R_{n_1}}, \quad (4)$$

with $\mathbf{C}^{(n)} = \begin{cases} \mathbf{G}^T, & \text{for} \quad n=1, n_1=2 \\ \mathbf{G}, & \text{for} \quad n=2, n_1=1 \end{cases}$ and $K_n = I_n J_n$. These matrix representations of $\mathcal{Z}^{(1)}$ and $\mathcal{Z}^{(2)}$ are associated with a contraction of the first two modes ($k_n = (i_n - 1)J_n + j_n$, for $n = 1$ and 2). Defining the quantities

$$z_{k_1,r_2}^{(1)} = z_{i_1,j_1,r_2}^{(1)}, \quad \text{and} \quad z_{k_2,r_1}^{(2)} = z_{i_2,j_2,r_1}^{(2)}, \quad (5)$$

(1) can be rewritten as two nested PARAFAC models

$$x_{i_1,j_1,k_2} = \sum_{r_1=1}^{R_1} a_{i_1,r_1}^{(1)} b_{j_1,r_1}^{(1)} z_{k_2,r_1}^{(2)}, \quad (6)$$

and

$$x_{i_2,j_2,k_1} = \sum_{r_2=1}^{R_2} a_{i_2,r_2}^{(2)} b_{j_2,r_2}^{(2)} z_{k_1,r_2}^{(1)}, \quad (7)$$

with respective matrix factors $(\mathbf{A}^{(1)}, \mathbf{B}^{(1)}, \mathbf{Z}^{(2)})$ and $(\mathbf{A}^{(2)}, \mathbf{B}^{(2)}, \mathbf{Z}^{(1)})$, where $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ are defined in (4).

It is worth noting that (6) and (7) are different contracted representations of the same fourth-order tensor $\mathcal{X}$ defined in (1), corresponding to two different ways of nesting the third-order PARAFAC models (2) and (3) into a single one. These two nested PARAFAC models (6) and (7) containing the full information of the original tensor model (1), admit the following matrix representations

$$\mathbf{X}_{J_1K_2 \times I_1} = (\mathbf{B}^{(1)} \diamond \mathbf{Z}^{(2)}) \mathbf{A}^{(1)T} \in \mathbb{C}^{J_1K_2 \times I_1}, \quad (8)$$

$$\mathbf{X}_{J_2K_1 \times I_2} = (\mathbf{B}^{(2)} \diamond \mathbf{Z}^{(1)}) \mathbf{A}^{(2)T} \in \mathbb{C}^{J_2K_1 \times I_2}, \quad (9)$$

which can be exploited to alternately estimate the matrix factors $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ using a two-step ALS algorithm, as will be shown in Section IV.

Uniqueness conditions: Application of the Kruskal's condition [10] allows concluding that the nested PARAFAC models (6) and (7) are essentially unique, i.e. their factor matrices are unique up to column permutation and scaling, if

$$k_{\mathbf{A}^{(n)}} + k_{\mathbf{B}^{(n)}} + k_{\mathbf{Z}^{(n_1)}} \geq 2R_n + 2, \quad \text{for} \quad (n, n_1) \in \{(1,2), (2,1)\} \tag{10}$$

where $k_{\mathbf{X}}$ denotes the Kruskal-rank (also called k-rank) of $\mathbf{X}$, corresponding to the largest integer $k_{\mathbf{X}}$ such that every set of $k_{\mathbf{X}}$ columns of $\mathbf{X}$ is independent.

III. System Model

Let M_t and M_r denote, respectively, the number of transmit and receive antennas in the considered multiple input multiple output (MIMO) communication system. At the transmitter, orthogonal frequency division multiplexing (OFDM) is used. We consider a group of F neighboring subcarriers across which the channel is assumed constant. A time-slotted transmission is considered, where each time-slot spans K symbol periods. If the channel is constant over a block time corresponding to T time-slots, the frequency-domain version of the discrete-time baseband received signal[1] in absence of noise can be written as

$$\mathbf{X}_{t,f} = \mathbf{H}\mathbf{U}_{t,f} \in \mathbb{C}^{M_r \times K}, \tag{11}$$

where $\mathbf{X}_{t,f}$ is the received signal matrix and $\mathbf{U}_{t,f} \in \mathbb{C}^{M_t \times K}$ is the complex space-time code matrix associated with the t-th time-slot and f-th subcarrier, with $E[\text{trace}(\mathbf{U}_{t,f}\mathbf{U}_{t,f}^H)] = KM_t$, $t = 1, \ldots, T$, $f = 1, \ldots, F$. The channel matrix $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ has i.i.d *CN*(0,1) entries, with $E[\text{trace}(\mathbf{H}\mathbf{H}^H)] = M_t M_r$. The transmitted signal power is normalized so that the signal-to-noise (SNR) ratio at each receive antenna is independent of the number of used transmit antennas. Let $\mathbf{s}_t \in \mathbb{C}^{M_t \times 1}$ denote the t-th transmitted symbol vector, satisfying $E[\text{trace}(\mathbf{s}_t^H \mathbf{s}_t)] = M_t$. The proposed STF encoding which defines $\mathbf{U}_{t,f}$ consists of two operations and is now detailed.

First, the symbol vector $\mathbf{s}_t$ is linearly precoded across M_t transmit antennas and F subcarriers using a set of frequency-dependent constellation rotation (CR) matrices $\{\boldsymbol{\Theta}_1, \ldots, \boldsymbol{\Theta}_F\}$. The (t, f)-th precoded symbol vector is denoted by $\mathbf{z}_{t,f} \doteq \boldsymbol{\Theta}_f \mathbf{s}_t$. The f-th CR matrix $\boldsymbol{\Theta}_f$ is chosen as $\boldsymbol{\Theta}_f \doteq \mathbf{G}\text{diag}(\mathbf{a}_f) \in \mathbb{C}^{M_t \times M_t}$, where $\mathbf{G} \in \mathbb{C}^{M_t \times M_t}$ is an M_t-point inverse discrete Fourier transform (DFT) matrix and

[1]The MIMO-OFDM system is modeled by set of F parallel flat-fading MIMO channels under the assumption of subcarrier orthogonality.

$\mathbf{a}_f \in \mathbb{C}^{M_t \times 1}$ is the f-th CR vector that performs successive phase rotations on the symbol vector $\mathbf{s}_t$ to be transmitted by the f-th subcarrier. For $F = 1$, optimized choices for the CR matrix $\boldsymbol{\Theta}$ are discussed in [11] for a given number of antennas and modulation type. Here, where $F \geq 1$, a suboptimal frequency-domain extension of constellation rotation is proposed. This feature induces a multilinear structure to the transmitted signal and is exploited by the proposed receiver as will be shown later.

The second operation "diagonally" encodes the CR precoded symbol vector $\mathbf{z}_{t,f}$ across K symbol periods using a coding matrix $\mathbf{C} \in \mathbb{C}^{K \times M_t}$ as follows [1]:

$$\mathbf{U}_{t,f} = \text{diag}\big(\mathbf{z}_{t,f}\big)\mathbf{C}^T. \tag{12}$$

Substituting $\mathbf{z}_{t,f} = \boldsymbol{\Theta}_f \mathbf{s}_t = \mathbf{G}\text{diag}(\mathbf{a}_f)\mathbf{s}_t$ into (12), we can write (11) as:

$$\mathbf{X}_{t,f} = \mathbf{H}\text{diag}\big(\mathbf{G}\text{diag}(\mathbf{a}_f)\mathbf{s}_t\big)\mathbf{C}^T. \tag{13}$$

Choice of $\mathbf{C}$ *and* $\mathbf{a}_f$*'s*: Along the lines of [1], we choose $\mathbf{C}$ as a Vandermonde matrix with generators $e^{j2\pi(m-1)/M_t}$, $m = 1, \ldots, M_t$, meaning that its (k,m)-th entry is given by $c_{k,m} \doteq e^{j2\pi(k-1)(m-1)/M_t}$. Additionally, we choose $\mathbf{a}_f \doteq [1, e^{j2\pi(f-1)/M_t}, \ldots, e^{j2\pi(f-1)(M_t-1)/M_t}]^T \in \mathbb{C}^{M_t \times 1}$, $f = 1, \ldots, F$. Although suboptimal, this choice ensures good channel and symbol identifiability properties, which is beneficial from the receiver design viewpoint. The code rate is given by $(\frac{M_t}{KF})\log_2(\mu)$, where μ is the modulation cardinality.

Note that the received signal model (13), associated with the t-th time-slot and f-th subcarrier, defines a matrix slice $\mathbf{X}_{t,f} \in \mathbb{C}^{M_r \times K}$ of the fourth-order tensor $\mathcal{X} \in \mathbb{C}^{M_r \times K \times T \times F}$ given by

$$\mathbf{X}_{t,f} = \mathbf{H}\text{diag}(\mathbf{G}\text{diag}(\mathbf{A}_{f.})\mathbf{s}_t)\mathbf{C}^T, \tag{14}$$

where we have defined $\mathbf{A}_{f.} \doteq \mathbf{a}_f$, i.e. $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_F]^T \in \mathbb{C}^{F \times M_t}$. This tensor $\mathcal{X}$ can be written elementwise as

$$x_{m_r,k,t,f} = \sum_{r_1=1}^{M_t}\sum_{r_2=1}^{M_t} h_{m_r,r_1} c_{k,r_1} s_{t,r_2} a_{f,r_2} g_{r_1,r_2}. \tag{15}$$

Proof: Denoting by $\mathbf{D} = \text{diag}(\mathbf{G}\text{diag}(\mathbf{A}_{f.})\mathbf{s}_t)$ the diagonal matrix that contains the space-frequency precoded signals to be time spread before transmission, the (m_r, k)-th entry of $\mathbf{X}_{t,f}$ is given by

$$x_{m_r,k,t,f} = \sum_{r_1=1}^{M_t} h_{m_r,r_1} c_{k,r_1} d_{r_1,r_1}, \tag{16}$$

with

$$d_{r_1,r_1} = \sum_{r_2=1}^{M_t} s_{t,r_2} a_{f,r_2} g_{r_1,r_2}. \tag{17}$$

Replacing (17) into (16) gives (15). ■

Comparing (15) with (1), we can conclude that the received signal tensor $\mathcal{X}$ satisfies two nested PARAFAC models, with the following correspondences

$$(I_1, I_2, J_1, J_2, R_1, R_2) \leftrightarrow (M_r, T, K, F, M_t, M_t), \tag{18}$$

$$(\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{B}^{(1)}, \mathbf{B}^{(2)}, \mathbf{G}) \leftrightarrow (\mathbf{H}, \mathbf{S}, \mathbf{C}, \mathbf{A}, \mathbf{G}). \tag{19}$$

Using the correspondences (18)-(19), the matrix representations (8)-(9) become

$$\mathbf{X}_{KTF\times M_r} = (\mathbf{C}\diamond(\mathbf{S}\diamond\mathbf{A})\mathbf{G}^T)\mathbf{H}^T, \tag{20}$$

$$\mathbf{X}_{FM_rK\times T} = (\mathbf{A}\diamond(\mathbf{H}\diamond\mathbf{C})\mathbf{G})\mathbf{S}^T. \tag{21}$$

Particular cases: The nested PARAFAC models satisfied by the D-KRSTF coding scheme reduce to two different PARAFAC models for $F=1$ and $K=1$, respectively. More specifically, from Eq. (20) we obtain the two following cases: i) For $K=1$, we have $\mathbf{X}_{TF\times M_r} = (\mathbf{S}\diamond\mathbf{A})(\mathbf{H}\mathbf{G})^T$, which represents a Khatri-Rao space-frequency (KRSF) coding model; ii) For $F=1$, we have $\mathbf{X}_{KT\times M_r} = (\mathbf{C}\diamond(\mathbf{S}\mathbf{G}^T))\mathbf{H}^T$, representing a Khatri-Rao space-time (KRST) coding model.

Uniqueness conditions: Due to its Fourier structure, $\mathbf{G}$ is full rank, and we have $k_{\mathbf{Z}^{(n)}} = k_{\mathbf{A}^{(n)}\diamond\mathbf{B}^{(n)}}$ in the uniqueness conditions (10). Applying the correspondences (18)-(19), these conditions become $k_{\mathbf{H}} + k_{\mathbf{C}} + k_{\mathbf{S}\diamond\mathbf{A}} \geq 2M_t + 2$ and $k_{\mathbf{S}} + k_{\mathbf{A}} + k_{\mathbf{H}\diamond\mathbf{C}} \geq 2M_t + 2$. Taking into account the Vandermonde structure of $\mathbf{A}$ and $\mathbf{C}$, we can conclude that these matrices are also full rank. Moreover, due to the random nature of $\mathbf{H}$ and $\mathbf{S}$, we have $k_{\mathbf{H}} \geq 1$ and $k_{\mathbf{S}} \geq 1$. We are interested in cases where the Khatri-Rao products $\mathbf{H}\diamond\mathbf{C}$ and $\mathbf{S}\diamond\mathbf{A}$ are full column-rank. In these cases, Kruskal's conditions then become

$$\min(M_r, M_t) + \min(K, M_t) \geq M_t + 2, \tag{22}$$

$$\min(T, M_t) + \min(F, M_t) \geq M_t + 2. \tag{23}$$

These conditions show that the numbers M_r of receive antennas and K of symbol periods play a symmetrical role, as it is the case also for the numbers T of time slots and F of subcarriers. Note that there are four situations in which both $\mathbf{H}\diamond\mathbf{C}$ and $\mathbf{S}\diamond\mathbf{A}$ are full column-rank. Assuming $M_t \geq 2$, we obtain the following practical corollaries:

- If $\mathbf{H}$ and $\mathbf{S}$ (or $\mathbf{A}$) are full column-rank, then $K \geq 2$ symbol periods and $F \geq 2$ subcarriers (or $T \geq 2$ time-slots) ensure uniqueness;
- If $\mathbf{C}$ and $\mathbf{A}$ (or $\mathbf{S}$) are full column-rank, then $M_r \geq 2$ receive antennas and $T \geq 2$ time-slots (or $F \geq 2$ subcarriers) ensure uniqueness;

IV. Semi-Blind Receiver

Assuming that the coding matrices $(\mathbf{A}, \mathbf{C}, \mathbf{G})$ are known at the receiver, we exploit (20)-(21) to derive a two-step alternating least squares algorithm (ALS) that alternately minimizes the following conditional LS criteria

$$\min_{\mathbf{H}} \|\mathbf{X}_{KTF\times M_r} - (\mathbf{C}\diamond(\hat{\mathbf{S}}_{it-1}\diamond\mathbf{A})\mathbf{G}^T)\mathbf{H}^T\|_F^2, \tag{24}$$

$$\min_{\mathbf{S}} \|\mathbf{X}_{FM_rK\times T} - (\mathbf{A}\diamond(\hat{\mathbf{H}}_{it}\diamond\mathbf{C})\mathbf{G})\mathbf{S}^T\|_F^2, \tag{25}$$

which gives the following two-step ALS to jointly estimate the channel and symbol matrices.

Initialization: Set $it = 0$; Randomly initialize $\hat{\mathbf{S}}_0$.

1) $it = it + 1$;
2) Compute the LS estimate of $\mathbf{H}$:

$$\hat{\mathbf{H}}_{it}^T = \left[\mathbf{C} \diamond (\hat{\mathbf{S}}_{it-1} \diamond \mathbf{A})\mathbf{G}^T\right]^\dagger \mathbf{X}_{KTF \times M_r}$$

3) Compute the LS estimate of $\mathbf{S}$:

$$\hat{\mathbf{S}}_{it}^T = \left[\mathbf{A} \diamond (\hat{\mathbf{H}}_{it} \diamond \mathbf{C})\mathbf{G}\right]^\dagger \mathbf{X}_{FM_rK \times T}$$

4) Repeat steps (1)-(3) until convergence.

After convergence, the scaling factors affecting the estimated symbol and channel matrices can be eliminated by using M_t known pilot symbols (one per transmit antenna) inserted in the first time-slot, meaning that the first row $\mathbf{S}_{1.}$ of the symbol matrix is known. The final estimate of the symbol matrix is obtained by simply re-scaling each column of the estimated symbol matrix: $\hat{\mathbf{S}}_{final} = \hat{\mathbf{S}}[\text{diag}(\hat{\mathbf{S}}_{1.})]^{-1}\text{diag}(\mathbf{S}_{1.})$. Consequently, the final estimate of the channel matrix is then given by $\hat{\mathbf{H}}_{final}^T = \left[\mathbf{C} \diamond (\hat{\mathbf{S}}_{final} \diamond \mathbf{A})\mathbf{G}^T\right]^\dagger \mathbf{X}_{KTF \times M_r}$.

Identifiability: The channel and symbol matrices are estimated by alternately minimizing the conditional LS criteria (24) and (25), with respect to $\mathbf{H}$ and $\mathbf{S}$, respectively. Uniqueness of these LS estimates requires that $\mathbf{C} \diamond (\mathbf{S} \diamond \mathbf{A})\mathbf{G}^T$ and $\mathbf{A} \diamond (\mathbf{H} \diamond \mathbf{C})\mathbf{G}$ be full column-rank, implying $FT \min(K, M_r) \geq M_t$. As for the uniqueness issue, this condition puts in evidence the symmetrical and complementary roles played by the frequency (F) and time (T) dimensions, on one hand, and the space (M_r) and time (K) dimensions, on the other hand, for jointly ensuring the identifiability of the symbol and channel matrices.

V. Simulation Results and Discussion

Next, we discuss the simulated performance of the proposed semi-blind receiver in terms of bit-error-rate (BER) and normalized mean square error (NMSE) of channel estimation. Each BER curve is an average of 10^4 Monte Carlo runs, each one representing a different channel realization, the coefficients of which are drawn from an i.i.d. complex-valued Gaussian generator. At each run, the transmitted data are drawn from PSK-type encoded symbols, and the additive noise power is generated according to the measured signal-to-noise ratio (SNR)= $10\log_{10}(\|\mathbf{X}_{KTF \times M_r}\|_F^2 / \|\mathbf{V}\|_F^2)$, where $\mathbf{V} \in \mathbb{C}^{KTF \times M_r}$ is the additive noise matrix, whose entries are modeled as zero-mean circularly symmetric complex Gaussian random variables.

In Figure 1, the BER and NMSE performances of D-KRSTF coding are compared with those of KRST and KRSF coding (particular cases of the proposed system), corresponding to pure time and frequency spreading, respectively [1]. We are particularly interested in configurations with $M_r < M_t$, where $M_r = 1$ or 2. This assumption is reasonable in the downlink of mobile communication systems where the number of antennas at the mobile terminal is limited due to physical space constraints. For D-KRSTF coding, the receiver algorithm proposed in Section IV is used. For KRST/KRSF coding, we have used a two-step ALS receiver derived from the resulting PARAFAC model [1]. The fixed settings are $M_t = 2$, $T = 5$ and 4-PSK. For $M_r = 1$, notice that D-KRSTF with $(K, F) = (2, 2)$ trades rate for diversity and significantly outperforms KRST and KRSF with $K = 2$ and $F = 2$, respectively. For $M_r = 2$, it can be seen that D-KRSTF with $(K, F) = (2, 2)$ offers a remarkable improvement over KRST under the same transmission rate. An extra improvement is obtained by doubling the value of K, i.e. using $(K, F) = (4, 2)$ at half transmission rate. Regarding the NMSE of the estimated channel, we notice that a more accurate channel estimation is achieved with D-KRSTF. The NMSE performance improves by increasing M_r and/or K.

In Figure 2, we draw comparisons of D-KRSTF with competing tensor-based space-time (ST) schemes operating at a code rate of 2. The fixed parameters are $M_t = M_r = 2$, $K = 3$, $F = 2$, $T = 5$ and 8-PSK. More precisely, we consider the following schemes: KRST [1], CONFAC-ST [4], and PARATUCK2-ST [5]. For the CONFAC-ST and PARATUCK2-ST schemes, the allocation matrices are fixed to the following structures:

$$\boldsymbol{\Psi}_{\text{CONFAC}} = \boldsymbol{\Phi}_{\text{CONFAC}} = \mathbf{I}_2 \otimes \mathbf{1}_2^T, \quad \boldsymbol{\Omega}_{\text{CONFAC}} = \mathbf{1}_2^T \otimes \mathbf{I}_2,$$

$$\boldsymbol{\Psi}_{\text{PARATUCK2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \boldsymbol{\Phi}_{\text{PARATUCK2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

The trilinear STF coding scheme [3] and the recently proposed PARATUCK2-STF scheme [7] are also included in our comparisons. For all of tensor-based STF schemes, the code rate is fixed to 1, and a two-step ALS receiver is used for joint channel and symbol estimation. This figure also shows the performance of the coherent maximum likelihood (ML) decoder for the D-KRSTF scheme, which assumes perfect channel state information (CSI). First, it can be seen that D-KRSTF outperforms the competing tensor-based ST schemes due to the extra degrees of freedom added by the frequency spreading dimension, which provides an additional coding gain. Compared to other tensor-based STF schemes, the proposed approach also presents a superior BER performance. It is worth mentioning that D-KRSTF has a simpler coding structure compared to trilinear STF and PARATUCK2-STF, although the latter affords a time-frequency allocation of data streams. In particular, PARATUCK2-STF has stronger constraints on the value chosen for P and involves a more computationally complex receiver processing.

In conclusion, our results confirm the merits of the D-KRSTF transceiver which arise by jointly using space, time and frequency domains for coding (at the transmitter) and, consequently, by exploiting two nested PARAFAC models for semi-blind symbol/channel estimation (at the receiver).

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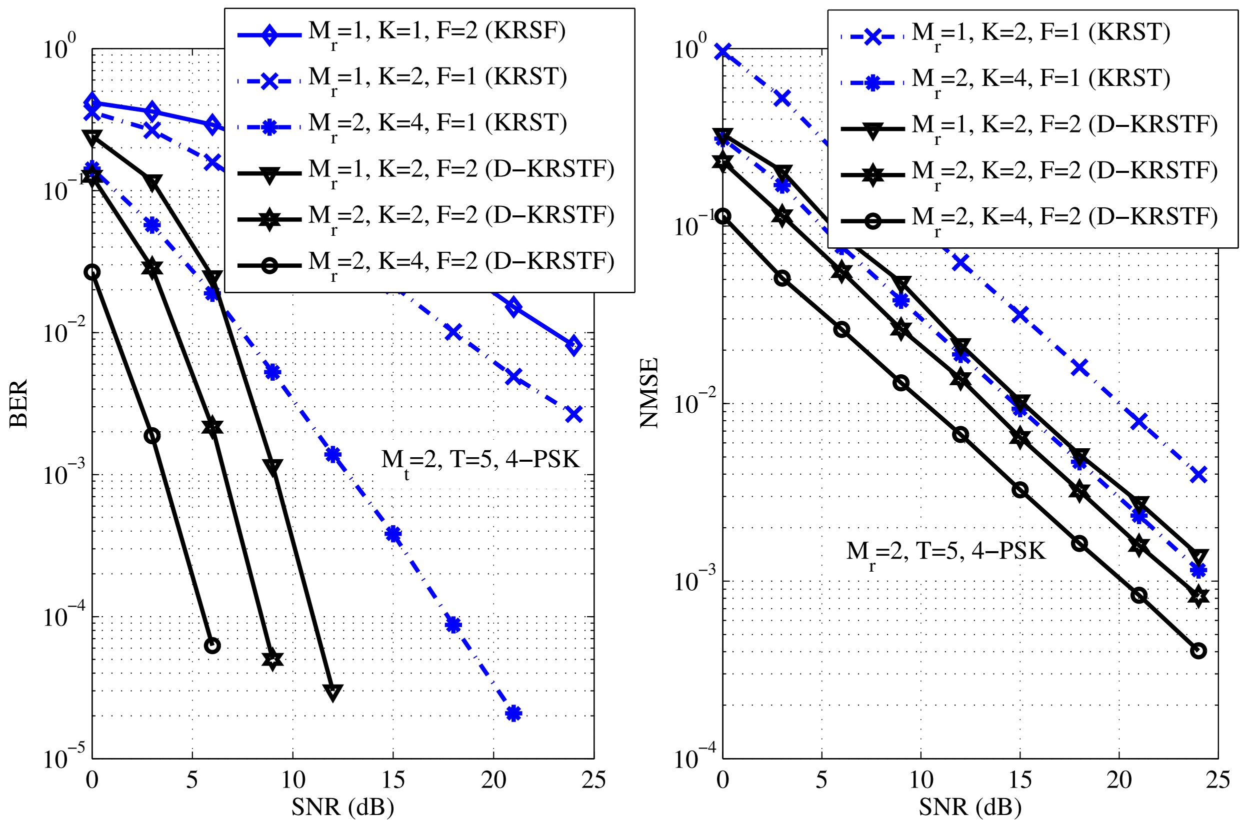

Fig. 1. BER and NMSE performances of D-KRSTF, KRST, and KRSF.

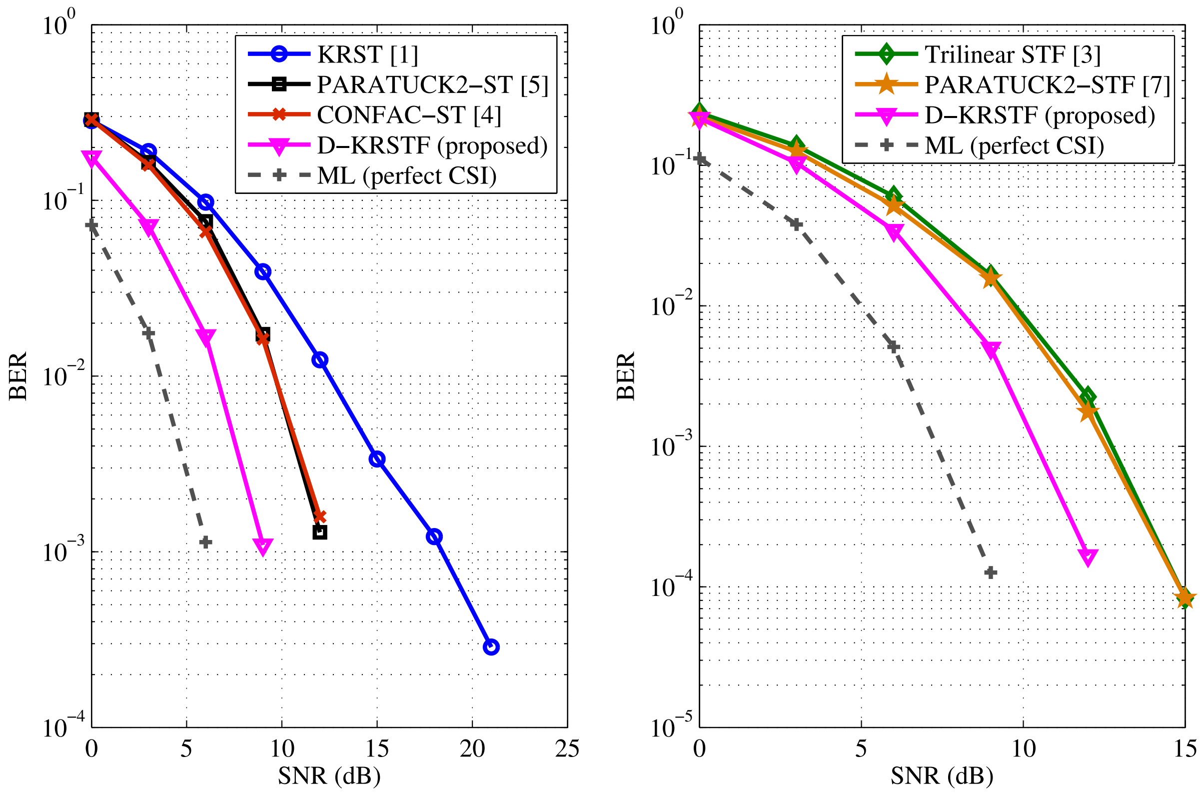

Fig. 2. Comparison with competing tensor-based solutions.